1 Introduction

The goal of this project is to investigate the following question: “Why is it that I get more snow on my windshield when my car is stopped at a light than when it’s moving, but I get more rain on my windshield when it’s moving than when it’s stopped?” The fluid mechanics topic most applicable to explaining this phenomenon is an analysis of the particle dynamics for snow and rain.

2 Problem Model

2.1 Assumptions

Modeling the problem requires defining a simplified system of the vehicle, the surrounding fluid (air), and the snow or rain particles. In order to standardize the fluid application of two different particles, the flux of snowfall and rainfall must be equal. Because the area of the car is constant, an equal flux ensures that the same number of particles have an equal opportunity to collide with the car. The rain and snow are assumed to have no horizontal velocity component, and particle collisions are neglected. The density \(D_{\text{air}} \ll D_{\text{particles}}\) and particle composition (e.g. dust) of the surrounding air are also neglected. Finally, there is no actual wind ensuring that the flow of air around the car is equivalent to the relative velocity \(V\) of the forward moving vehicle.

2.2 Snow and Rain as Particles

The structure of a snowflake and a raindrop are different. A snowflake is comprised of a crystalline structure of ice \([3]\), but a raindrop is liquid water. Both have different masses and densities. The mass of an average snowflake is 3 mg \([2]\); a raindrop is 70 to 100 mg \([1]\). A raindrop is far denser than a snowflake, not only because liquid water is denser than ice, but also because the crystal structure of snow occupies a larger volume. The density of water is 1000 kg/m\(^3\) while the density of snow is approximately 100 kg/m\(^3\) \([4]\).

2.3 Stokes Number

Stokes Number \(Sk\) is a dimensionless parameter that describes a particle’s flow in a particular fluid, for example snow in air. Stokes number is determined by the ratio of the relaxation time of the particle \(\tau\), a characteristic dimension of the obstacle obstructing fluid flow \(D\) and the fluid’s velocity \(V\):

\[
Sk = \frac{\tau V}{D}
\]

The relaxation time, or particle response time, is the time required for particle to respond to a change of the fluid’s velocity such that when the surrounding fluid velocity changes, as it might when encountering an object, the particle follows the fluid and flows around the object. Relaxation time is calculated from the diameter of the particle \(d_p\), particle density \(\rho_p\), and dynamic viscosity of the surrounding fluid \(\mu\) \([7]\):
\[ \tau = \frac{d^2 \rho p}{18 \mu} \]  

(2)

When the equations are combined and the particle diameter is substituted for the characteristic dimension, the Stokes Number for a particle is calculated as [9]:

\[ Sk = \frac{\rho d^2 V}{18 \mu D} \]  

(3)

A value of 20 m/s (≈ 45 mph) is used as a model velocity for the car and resultant airflow. The car is assumed to have a characteristic dimension of \( D = 1 \) m. Air at 0 degrees Celsius has a dynamic viscosity of \( 1.71 \times 10^{-5} \text{ Ns/m}^2 \). Stokes Number for a raindrop \( Sk = 584 \) is calculated from a diameter of \( d_p = 0.003 \text{ m} = 3 \text{ mm} \) and a density of \( \rho = 1000 \text{ kg/m}^3 \). Stokes Number \( Sk = 58 \) for snow is calculated using the same diameter and a density of \( \rho = 100 \text{ kg/m}^3 \).

Generally, a Stokes Number much greater than 1 (\( Sk >> 1 \)) describes particles that remain unaffected by a fluid velocity change and continue their original trajectory, colliding with the object rather than flowing around it. If \( Sk << 1 \), the particle will follow the fluid’s velocity and move around the object without colliding.

Density is the major difference between the raindrop and snowflake. The Stokes Number of a snowflake is much lower than a raindrop, so a snowflake is expected to follow a fluid better than a raindrop. The relation of Stokes Number and density is shown in figure (1).

![Figure 1: A plot displaying the relationship between Stokes Number and particle density.](image)

3 Understanding the Application

To understand the problem, it is best to think of two cases. First is that of a stationary car with no resultant airflow. This is analogous to the car at a stoplight. In this case, raindrops and snowflakes collide with the perpendicular surfaces of the car. Because it was assumed that raindrops and snowflakes have equal flux, the same number of raindrops and snowflakes collide with the car. This is further supported by the calculation of Stokes Number of any particle for a non-moving car, because \( V = 0 \) m/s, Stokes Number is then 0. This
indicates that the particles will follow the flow of the fluid and since there is no actual wind present the particles settle on the windshield.

The second case, is that of a moving car, in which a moving car creates a resultant airflow with velocity $V$. As previously discussed, due to different densities, Stokes Number for rain particles is greater than snow particles. The snow particles will follow the resultant airflow better than rain. This is shown in figure (2). Additionally, forward movement of the vehicle results in more rain collisions when moving because additional surfaces of the car now collide with rain.

![Diagram of car with resultant airflow](image)

Figure 2: A diagram of the car with resultant airflow. Snow particles with a low Stokes Number are carried by the moving fluid. Rain particles with a high Stokes Number settle onto the windshield. If there was no resultant fluid flow, both particles would settle.

Therefore, differing Stokes Number provides support for the conclusion that more snow will collide with a stationary car than a moving car. Forward velocity allows more rain to collide with a moving car than a stationary car.

4 Other Factors

Although Stokes Number provides the primary model for problem analysis, it is useful to consider a few other factors that might play a roll in the model.

Density alone does not affect the amount of rain or snow that collides with a stationary surface during a given amount of time. Structural differences result in different drag coefficients. Rain is a fluid that deforms when a sheering force is applied, in this case the drag force from wind velocity. The best model for a rain drop is that of a streamlined body with a drag coefficient $C_D = 0.04$ [5]. Contrary to the rain structure, the crystalline structure of the snowflake provides structural support that prevents a falling snowflake from deforming as it falls. This results in a higher drag coefficient for a snowflake, best modeled as a solid sphere. Different drag coefficients result in different behavior in a fluid, usually related to the particle’s terminal velocity [8].

The diameter of an average snowflake and raindrop are also different. Raindrops vary between 1 and 3 $mm$ in diameter, but snowflakes can be as large as 10 $cm$ [6].
5 References


